AQA

A-LEVEL
Mathematics

MD02 – Decision 2
Mark scheme

6360

June 2018

Version/Stage: 1.0 Final

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

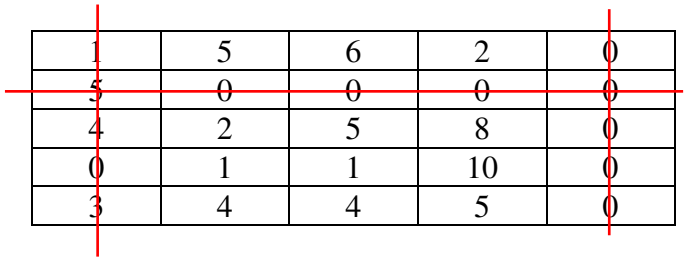
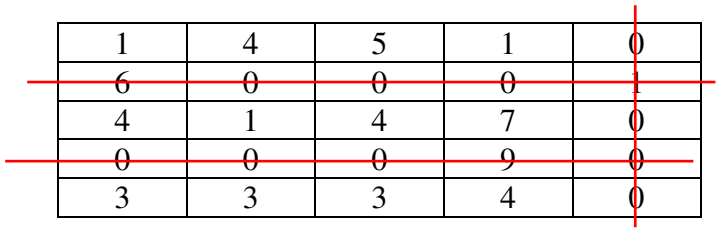
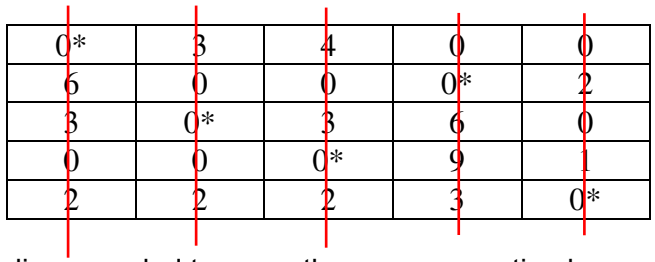
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline 6 & 11 & 7 & 2 & ** \\ \hline 10 & 6 & 1 & 0 & ** \\ \hline 9 & 8 & 6 & 8 & ** \\ \hline 5 & 7 & 2 & 10 & ** \\ \hline 8 & 10 & 5 & 5 & ** \\ \hline \end{array}$	**B1** **B1**	**2**	All elements x replaced with $k-x,\ k \geq 19$ Extra column introduced with all values being the same non-negative number may be seen in (b)
(b)	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline 1 & 5 & 6 & 2 & 0 \\ \hline 5 & 0 & 0 & 0 & 0 \\ \hline 4 & 2 & 5 & 8 & 0 \\ \hline 0 & 1 & 1 & 10 & 0 \\ \hline 3 & 4 & 4 & 5 & 0 \\ \hline \end{array}$	**M1** **A1**		Column reduction with 4 or more columns correct All columns correct and three lines used to cover zeros (may have vertical line in 1st col, see alt)
	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline 0 & 4 & 5 & 1 & 0 \\ \hline 5 & 0 & 0 & 0 & 1 \\ \hline 3 & 1 & 4 & 7 & 0 \\ \hline 0 & 1 & 1 & 10 & 1 \\ \hline 2 & 3 & 3 & 4 & 0 \\ \hline \end{array}$	**dM1** **A1**		Augmenting (by 1) with 4 or more rows/columns correct All correct with three lines drawn to cover zeros
	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline 0^* & 3 & 4 & 0 & 0 \\ \hline 6 & 0 & 0 & 0^* & 2 \\ \hline 3 & 0^* & 3 & 6 & 0 \\ \hline 0 & 0 & 0^* & 9 & 1 \\ \hline 2 & 2 & 2 & 3 & 0^* \\ \hline \end{array}$	**dM1**		Augmenting (by 1) with 4 or more rows/columns correct
	Five lines needed to cover the zeros, so optimal allocation found	**A1**		All correct with five lines drawn to cover zeros and 'optimal' seen
	Tom will not be selected for the team OE	**A1**	**7**	CSO
	Total		**9**	

Notes: Candidate may write answer to (a) in (b), do not penalise, give benefit of doubt to candidate
(b) need to see (at least) 'optimal' for **A1**
Statement about Tom needs to be in context, not just 'Tom'

Q1	Solution	Mark	Total	Comment
(b)	**Alternative**  1 5 6 2 0 5 0 0 0 0 4 2 5 8 0 0 1 1 10 0 3 4 4 5 0	**M1** **A1**		Column reduction with 4 or more columns correct All columns correct and three lines used to cover zeros
	 1 4 5 1 0 6 0 0 0 1 4 1 4 7 0 0 0 0 9 0 3 3 3 4 0	**dM1** **A1**		Augmenting (by 1) with 4 or more rows/columns correct All correct with three lines drawn to cover zeros
	 0* 3 4 0 0 6 0 0 0* 2 3 0* 3 6 0 0 0 0* 9 1 2 2 2 3 0*	**dM1**		Augmenting (by 1) with 4 or more rows/columns correct
	Five lines needed to cover the zeros, so optimal allocation found	**A1**		All correct with five lines drawn to cover zeros and 'optimal' seen
	Tom will not be selected for the team	**A1**	**7**	CSO
	Total		**9**	

<table>
<tr><th>Q2</th><th>Solution</th><th>Mark</th><th>Total</th><th>Comment</th></tr>
<tr><td>(a) (i)</td><td>$[18 + 20 + 25 + 87] = 150$</td><td>B1</td><td></td><td></td></tr>
<tr><td>(a) (ii)</td><td>$[51 + 14 – 5 – 3 + 87] = 144$</td><td>B1</td><td>2</td><td></td></tr>
<tr><td>(b)</td><td>[Max] flow ≤ their min from (a)</td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td>[Max] flow of gas [through the network of] pipes must be less than or equal to $144\ \text{cm}^3\ \text{s}^{-1}$ OE</td><td>A1</td><td>2</td><td>Including units</td></tr>
<tr><td>(c)(i)</td><td><table><tr><th>Arc</th><th>Forward</th><th>Backward</th></tr><tr><td>SA</td><td>29</td><td>0</td></tr><tr><td>AC</td><td>4</td><td>0</td></tr><tr><td>CF</td><td>17</td><td>5</td></tr><tr><td>FT</td><td>68</td><td>0</td></tr><tr><td>AD</td><td>1</td><td>6</td></tr><tr><td>DF</td><td>6</td><td>5</td></tr><tr><td>DC</td><td>15</td><td>0</td></tr><tr><td>DE</td><td>8</td><td>0</td></tr><tr><td>SB</td><td>32</td><td>0</td></tr><tr><td>BE</td><td>14</td><td>3</td></tr><tr><td>EG</td><td>4</td><td>10</td></tr><tr><td>GT</td><td>67</td><td>0</td></tr><tr><td>BD</td><td>0</td><td>4</td></tr><tr><td>DG</td><td>22</td><td>2</td></tr></table></td><td>M1

M1

A1</td><td>3</td><td>Correct at SA, AC, SB, and BE

Correct at CF, FT, EG, and GT

All correct</td></tr>
<tr><td>(c)(ii)</td><td>Modifying one feasible flow (both increasing and decreasing) correctly on the diagram

e.g.<table><tr><th>Path</th><th>Extra Flow</th></tr><tr><td>SACFT</td><td>4</td></tr><tr><td>SAD(CF, F or G)T</td><td>1</td></tr><tr><td>SBEGT</td><td>4</td></tr></table></td><td>B1

M1

A1

A1</td><td>4</td><td>

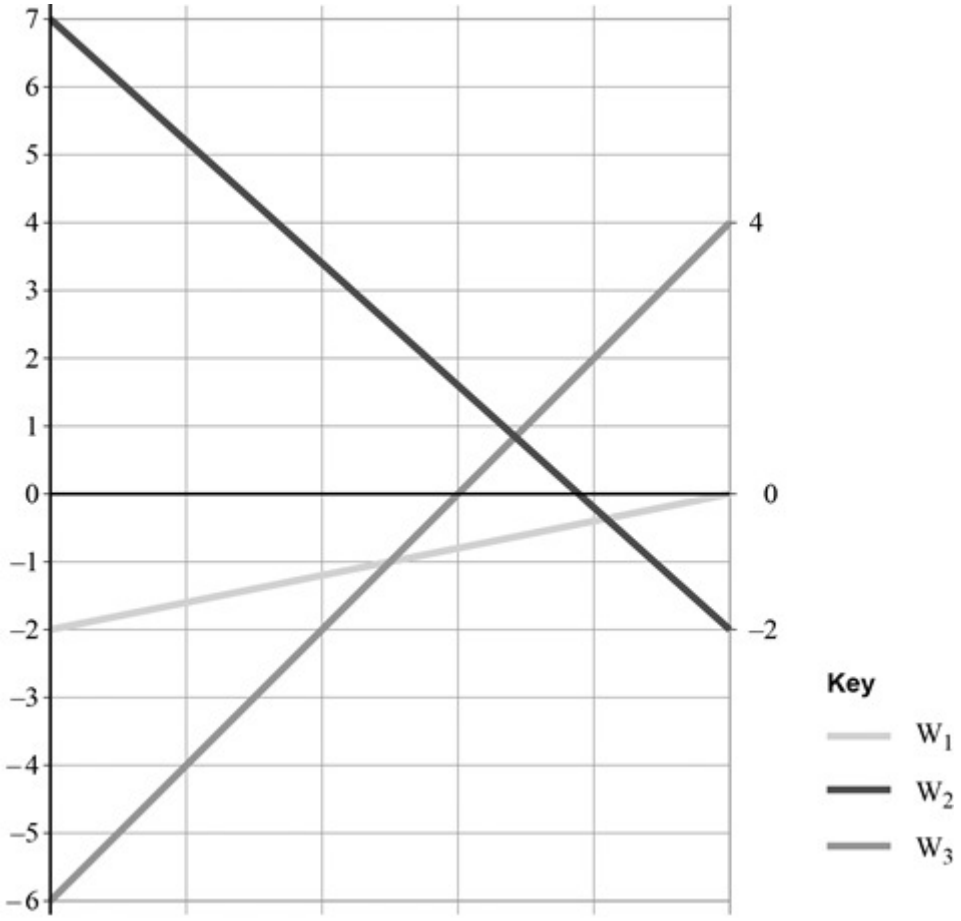
One correct path and extra flow in table

Two correct paths and extra flows in table

All correct</td></tr>
<tr><td>(c)(iii)</td><td>[Max. flow = $38 + 4 + 1 + 4 =$] 47 $[\text{cm}^3\ \text{s}^{-1}]$</td><td>B1</td><td>1</td><td></td></tr>
<tr><td></td><td>Total</td><td></td><td>12</td><td></td></tr>
</table>

Notes: **(b)** may see symbols for 'less than or equal to'
(c) If **M0** scored in (i), then candidate scores **B0** in (ii)

(c)(i) if **M0M0** scored, **SC1** for *SA, AC, CF, FT* or *SB, BE, EG, GT* all correct
(c)(ii) there are other obtuse possibilities, but any correct soln must have total(*SA…T)* = 5 and *SBEGT* = 4
Note: there is no path *SACD ..T* allowable

Q3	Solution	Mark	Total	Comment
(a)	Use of p and $1-p$ for John's strategies	**M1**		
	[If Winnie plays] / [Expected gain for John] $[W_1]$ / $0p + (-2)(1-p)$ $[= 2p - 2]$ $[W_2]$ / $-2p + 7(1-p)$ $[= 7 - 9p]$ $[W_3]$ / $4p + (-6)(1-p)$ $[= 10p - 6]$	**A1**		One correct expression
		A1		All three expressions correct
		**M1**		Three lines
		A1		All three **ruled** lines correct (-2 to 0, 7 to -2 and -6 to 4) with numbers on both vertical axes
	[Optimal/maximum at] $2p - 2 = 7 - 9p$	**A1**		
	$[p = 9/11]$			
	[Value of the game for John]: $[2 \times (9/11) - 2] = -4/11$ OE	**A1**	**7**	Must be exact

(b)

[If John plays]	[Expected gain for Winnie]			
$[\mathbf{J_1}]$	$0q + (-2)r + 4(1 - q - r)$ $[= 4 - 4q - 6r]$	**M1**		Either expression correct (including the use of exactly two probability variables)
$[\mathbf{J_2}]$	$-2q + 7r + (-6)(1 - q - r)$ $[= -6 + 4q + 13r]$			
$4 - 4q - 6r = -4 / 11$ $-6 + 4q + 13r = -4 / 11$		**A1F**		Sets the correct expressions equal to their value of the game from **(b)**
$q = 9/11$, $r = 2/11$		**A1**		Both values correct
Winnie plays: $\mathbf{W_1}$ [with probability] $9/11$ $\mathbf{W_2}$ [with probability] $2/11$ $\mathbf{W_3}$ [with probability] 0		**E1**	**4**	Must have all three probabilities

Alternative Solution

Winnie never plays $\mathbf{W_3}$ so

[If John plays]	[Expected gain for Winnie]			
$[\mathbf{J_1}]$	$0p + (-2)(1 - p)$ $[= -2 + 2p]$	**(M1)**		Either expression correct
$[\mathbf{J_2}]$	$-2p + 7(1 - p)$ $[= 7 - 9p]$			
$-2 + 2p = 7 - 9p$		**(A1)**		Sets the correct gain expressions equal to each other
$p = 9/11$		**(A1)**		
Winnie plays: $\mathbf{W_1}$ [with probability] $9/11$ $\mathbf{W_2}$ [with probability] $2/11$ $\mathbf{W_3}$ [with probability] 0		**(E1)**		Must have all three probabilities, but $\mathbf{W_3}$ may be stated as never played earlier in the solution

	Total		**11**	

Notes: Marks for (a) may be earned in a candidate's solution to (b). Consult team leader
(a) Each **A** mark is only dependent on the previous **M** mark eg **M1A1A0 M1A0A1** …

Q4	Solution					Mark	Total	Comment
	Stage	**State**	**From**	**Value**				
	1	*I*	*T*		475*			
		J	*T*		480*			
		K	*T*		475*			
	2	*D*	*I*	max(470, 475)	475*	**B1**		9 Values at Stage 2
		E	*I*	max(470, 475)	475*			
			J	max(465, 480)	480			
		F	*I*	max(495, 475)	495	**M1**		Using minimax – choosing at least two of EI, FJ or GK (PI)
			J	max(490, 480)	490*			
			K	max(495, 475)	495			
		G	*J*	max(485, 480)	485			
			K	max(480, 475)	480*			
		H	*K*	max(475, 475)	475*	**A1**		All **values** correct at Stage 2
	3	*A*	*D*	max(480, 475)	480*			
			E	max(515, 475)	515			
			F	max(490, 490)	490	**B1**		9 Values at Stage 3
		B	*E*	max(485, 475)	485			
			F	max(475, 490)	490	**dM1**		At least 7 values correct
			G	max(480, 480)	480*			
		C	*F*	max(490, 490)	490*	**A1**		All **values** correct at Stage 3
			G	max(500, 480)	500			
			H	max(495, 475)	495			
	4	*S*	*A*	max(465, 480)	480*	**B1**		3 Values at Stage 4
			B	max(470, 480)	480*			
			C	max(460, 490)	490	**A1**		All **values** correct at Stage 4
	S-B-G-K-T					**A1**		One correct route (not reversed)
	S-A-D-I-T					**A1**		2nd correct route (not reversed) AND no others
	[Longest Days Driving Time =] 480 minutes OE					**B1**	**11**	Must include units
					Total	**11**		
Notes: condone omission of max comparisons eg max(470, 475)								

<table>
<tr><th>Q5</th><th>Solution</th><th>Mark</th><th>Total</th><th>Comment</th></tr>
<tr><td>(a)</td><td>$a < 0$</td><td>B1</td><td>1</td><td></td></tr>
<tr><td>(b)</td><td>$\frac{b}{1/2} < \frac{c}{3/2}$</td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td>$3b < c$</td><td>A1</td><td>2</td><td>NMS 2/2</td></tr>
<tr><td>(c)(i)</td><td>$d \pm 3 \times 7/2$</td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td>$d + 21/2$ OE</td><td>A1</td><td>2</td><td>NMS 2/2</td></tr>
<tr><td>(c)(ii)</td><td><table><tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>value</th></tr><tr><td>1</td><td>0</td><td>0</td><td>$35 - 7a$</td><td>$-2a$</td><td>$10 + a$</td><td>0</td><td>$100 - 2ab$</td></tr></table></td><td>B1

M1

A1

B1</td><td>

4</td><td>P = 1, x = 0, y = 0 and u = 0 all correct

One of z, s or t correct

z, s and t all correct

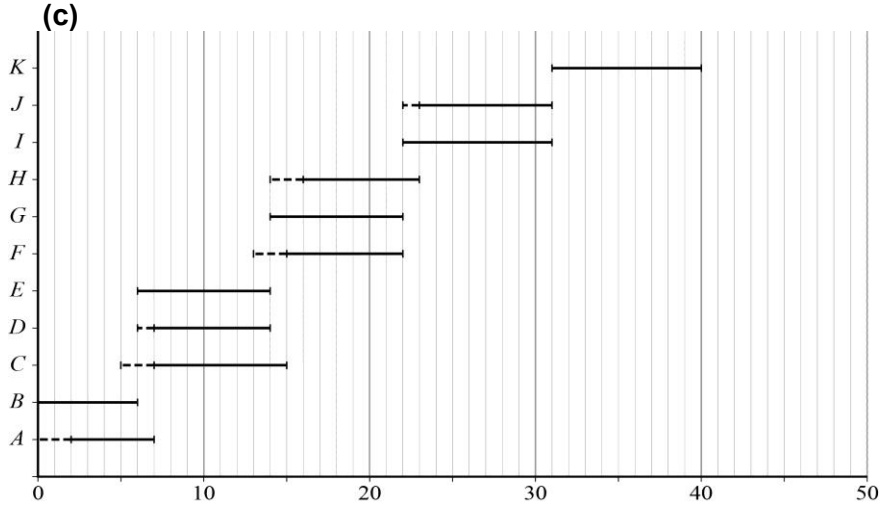
value = 100 – 2ab</td></tr>
<tr><td>(c)(iii)</td><td>$10 + a \geq 0$ OE</td><td>M1</td><td></td><td>$their\ t \geq 0$
PI by final answer</td></tr>
<tr><td></td><td>$-10 \leq a < 0$</td><td>A1</td><td>2</td><td>Allow $-10 \leq a$ AND $a < 0$</td></tr>
<tr><td></td><td>Total</td><td></td><td>11</td><td></td></tr>
</table>

Notes: **(b)** For **A1,** accept $b < \frac{1}{3}c$, or $\frac{1}{3}c > b$ or c > 3b BUT NOT $6b < 2c$

(c)(i)(ii) for both parts condone correct multiples for all marks eg $d/k + 21/2k$
(ii) condone omission of 'heading row', but be convinced
(iii) *their* $(10 + a) \geq 0$ seen anywhere in soln scores **M1**

<table>
<tr><th>Q6</th><th>Solution</th><th>Mark</th><th>Total</th><th>Comment</th></tr>
<tr><td>(a)</td><td>
<table>
<tr><th>Activity</th><th>Early</th><th>Late</th></tr>
<tr><td>A</td><td>0</td><td>7</td></tr>
<tr><td>B</td><td>0</td><td>6</td></tr>
<tr><td>C</td><td>5</td><td>15</td></tr>
<tr><td>D</td><td>6</td><td>14</td></tr>
<tr><td>E</td><td>6</td><td>14</td></tr>
<tr><td>F</td><td>13</td><td>22</td></tr>
<tr><td>G</td><td>14</td><td>22</td></tr>
<tr><td>H</td><td>14</td><td>23</td></tr>
<tr><td>I</td><td>22</td><td>31</td></tr>
<tr><td>J</td><td>22</td><td>31</td></tr>
<tr><td>K</td><td>31</td><td>40</td></tr>
</table>
</td><td>B1

B1</td><td>2</td><td>All early times correct

All late times correct</td></tr>
<tr><td>(b)</td><td>B-E-G-I-K</td><td>B1</td><td>1</td><td>Or reverse</td></tr>
<tr><td>(c)</td><td>
</td><td>M1

B1

A1</td><td>3</td><td>SCA, at least 10 labelled activities

Use of floats, at least three of A, C, D, F, H, J (either before or after activity)

All correct, including labelling and all floats before activity correct</td></tr>
<tr><td>(d)(i)</td><td>Reduce G by 5 days

Reduce both F and H by 3 days</td><td>E1

E1</td><td></td><td>Decrease G to 3 days

Decrease F and H both to 4 days</td></tr>
<tr><td>(d)(ii)</td><td>Cost of reducing to the minimum completion time
$(g \times 500) + (f \times 250) + (h \times 650)$
(with at least 2 of their f, g, h non-zero)

[£]5200

[£]5200 > [£]5000, (so the company with not make a profit from the bonus payment) so don't reduce (F, G, H)</td><td>M1

A1

E1F</td><td>5</td><td>Calculates cost of reducing durations by 'their' reductions in (d)(i)

CAO

Comparing their 5200 to 5000 PI and making a correct conclusion about whether to reduce durations</td></tr>
<tr><td></td><td>Total</td><td></td><td>11</td><td></td></tr>
<tr><td colspan="5">Notes: (c) For M1 all non-critical activities must be on a separate row
(d)(ii) eg if their answer is less than 5000, (the company would benefit from the bonus) so reduce E1F
Accept: 'Yes/No' plus valid reason in context</td></tr>
</table>

Q7	Solution	Mark	Total	Comment
(a)	Row Minima = $-2, -5, -6, -2$ Column Maxima = $-2, 6, -2, 7$	**M1**		Finds all row minima or all column maxima (may be seen on table.) Condone one slip
	Max(Row Minima) = -2 Min(Row Maxima) = -2	**A1**		Both correct (may be indicated, not stated from 8 correct values)
	Any two of AW, AY, DW and DY stated	**A1**		
	All four saddle points correct and no others stated	**A1**	**4**	AW, AY, DW and DY.
(b)	**Y** dominates **Z**	**B1**		
	as $\begin{pmatrix} 2x \\ x-3 \\ -2 \\ -2 \end{pmatrix} < \begin{pmatrix} 3 \\ -5 \\ 7 \\ 6 \end{pmatrix}$ for integer $x < -3$.	**E1**		**OE** statement with $x < -3$ mentioned/used
	D dominates **C**	**B1 dep**		Dependent on first B mark
	as $(x-1 \quad 1 \quad -2) \geq (x-3 \quad 3x-3 \quad -2)$ for integer $x < -3$.	**E1**		**OE** statement with $x < -3$ mentioned/used
	W (or **Y**) dominates **X**	**B1 dep**		Dependent on first and second B marks, and no other further dominances stated/used
	as $\begin{pmatrix} -2 \\ -3 \\ x-1 \end{pmatrix} < \begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix}$ for integer $x < -3$ $\left[or \begin{pmatrix} 2x \\ x-3 \\ -2 \end{pmatrix} < \begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix} \right]$	**E1**	**6**	**OE** statement with $x < -3$ mentioned/used
	Total		**10**	

Notes: (a) the final **A1A1** are independent of previous **A1**
the 'co-ordinates eg AW' must be stated not merely circling entries in the table for the final **A1A1**
If **M0** scored then **SC1** for two or more correct saddle points
(b) working may be seen on table
A candidate may subst $x = -4$ (instead of $x < -3$) and consider dominance as above. In this case all marks are available. (possibly by drawing a new amended matrix)
A candidate who subst $x = -3$ can score the **B** marks only